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Near the horizon of $5D$ black rings

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Abstract: For the five dimensional $\mathcal{N} = 2$ black rings, we study the supersymmetry enhancement and identify the global supergroup of the near horizon geometry. We show that the global part of the supergroup is $OSp(4^*|2) \times \mathrm{U}(1)$ which is similar to the small black string. We show that results obtained by applying the entropy function formalism, the c-extremization approach and the Brown-Henneaux method to the black ring solution are in agreement with the microscopic entropy calculation.

Keywords: Black Holes in String Theory, Supergravity Models

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1 Introduction and summary

Five dimensional supergravity is interesting from several points of view. Such a supergravity can be constructed by compactifying the eleven dimensional supergravity, on some six dimensional manifolds e.g. CY_3 or T^6. There are several supersymmetric solutions for the five dimensional supergravity that preserve either one half or all of the supersymmetry [1]. These solutions contain three kinds of black objects which are half-BPS known as black holes, black strings and black rings which their near horizon geometries are $AdS_2 \times S^3$, $AdS_3 \times S^2$ and $AdS_2 \times S^1 \times S^2$ respectively. Each of these solutions has a specific charge configuration. A black hole has only electric charges, a black string has only magnetic charges, while a black ring has both electric and magnetic charges. A nice review on these black solutions of $\mathcal{N} = 2$ five dimensional supergravity is [2].

Black ring is the first example of a black object with a non-spherical horizon topology and asymptotically flat geometry which carries angular momentum along the S^1 direction [3]. Furthermore, the existence of this solution implies that the black hole uniqueness theorems can not be extended to five dimensions, except in the static case [4]. The generalization of the uniqueness of black holes to five dimensions is studied in [5], where it is shown that the dipole charge appears in the first law of thermodynamics in the same manner as a global charge. Therefore there exist black objects with the same *global* charges

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but with different horizon topologies. Some other developments are listed in [6]–[11]. For a good review on black ring see [12].

In this paper we study some features of the supersymmetric large black rings in the five dimensional $\mathcal{N} = 2$ supergravity which have the non-zero classical horizon area. Large black rings are half-BPS and in the near horizon limit they exhibit supersymmetry enhancement [13]. We want to investigate the symmetry of the near horizon geometry of the supersymmetric black rings. For this purpose, we note that $\mathcal{N} = 2n$ supergravity in five dimensions with $8n$ real supercharges has an $\mathrm{Sp}(2n)$ R-symmetry group with the supersymmetry parameter ε^i, $i = 1, \ldots, 2n$, transforming as **2n** representation. Using this fact, one can solve for the supersymmetry spinor and calculate the global part of the superalgebra. Doing so, we show that the global part of the superalgebra is $OSp(4^*|2) \times \mathrm{U}(1)$, which is similar to the small black string obtained in [14].

The most important reason for investigating the supergroup of the near horizon geometry of the black objects is the AdS/CFT correspondence. AdS_2/CFT_1 correspondence is not well-defined yet in contrast to the higher dimensional cases (see for example [15]). Motivated by this phenomenon, the symmetry of the near horizon geometry of the small black hole solutions of $\mathcal{N} = 2, 4$ supergravity in five dimensions is studied in [16]. Lapan *et al* in [14], study the symmetry of the near horizon geometry of small black string solutions to investigate AdS_3/CFT_2 correspondence, which in principle gives some information about AdS_2/CFT_1 via dimensional reduction. Some recent results on the AdS_3/CFT_2 correspondence and the small black strings can be found in [17]. It seems that for studying AdS_2/CFT_1 from AdS_3/CFT_2, the black ring is a better starting point than the black string since the fibration of S^1 over AdS_2 is explicit.[1] Thus our study of the near horizon physics of the black ring might shed a new light on this subject.

An important feature of supersymmetric black objects is the attractor mechanism. The attractor mechanism determines the value of the scalar fields near the horizon independent of their asymptotic values, and also implies the enhancement of supersymmetry near the horizon [19]. Attractor mechanism as reformulated by Sen, which is called the "entropy function formalism", can be used to calculate the entropy of black holes with $AdS_2 \times X$ near horizon geometry in diverse dimensions [20]. In [21–23] the entropy function formalism is applied to black rings. As mentioned above, the near horizon geometry of the black ring solution is $AdS_2 \times S^1 \times S^2$, where S^1 is fibred nontrivially over AdS_2. This phenomenon as well as the Chern-Simons term in the five dimensional supergravity frustrates a direct application of the entropy function formalism in this case.[2] In fact there are two problems in applying the entropy function method to black rings. First, in the Wald formula [25] there is a derivative of the Lagrangian density with respect to the Riemann tensor components which for $AdS_2 \times X$ near horizon geometry has only one independent component.[3] But in

[1] The same property of black ring gives an opportunity to study the relation between $4D$ black holes and $5D$ black rings [18].

[2] The difficulty in incorporating the Chern-Simons term into the entropy function formalism is studied in [24].

[3] Interestingly, for the D1-D5-P black holes, where a similar problem is encountered, a generalization of

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the case of the black ring near horizon geometry, the Riemann tensor has four independent components since S^1 is fibred non-trivially over AdS_2.[4] Second, the Chern-Simons term in the Lagrangian density is not gauge invariant,[5] while in the entropy function formalism the gauge invariance of the Lagrangian density is assumed. We study the entropy function formalism for the black ring and explain how both of these problems can be resolved by dimensional reduction along the S^1. By such a dimensional reduction, the near horizon geometry reduces to $AdS_2 \times S^2$, which has only one relevant independent Riemann tensor component, and the Chern-Simons term becomes a sum of gauge invariant terms.

In [28], Kraus and Larsen introduced the c-extremization approach for obtaining the spacetime central charge of black objects with $AdS_3 \times Y$ near horizon geometry in a simple way. Although, the c-extremization is introduced for black objects with a globally AdS_3 component of the near horizon geometry, we show that by applying this method to the black ring which horizon geometry locally looks like $AdS_3 \times S^2$, one obtains results which are in agreement with the outcome of the entropy function formalism and microscopic calculations of the black ring entropy [11, 12, 29, 30].

We recalculate the microscopic entropy by the Kerr/CFT correspondence [31], which is intrinsically a generalization of Brown-Henneaux approach [32] to AdS/CFT correspondence [33]. Choosing an appropriate boundary condition we show that the asymptotic symmetry group of the near horizon of supersymmetric black ring contains a Virasoro algebra. The corresponding central charge equals the c-extremization result. By defining the Frolov-Thorne temperature [35] and using the Cardy formula we calculate the CFT entropy and show that it equals the Bekenstein-Hawking entropy.

The main results of this work are that in five dimensional $\mathcal{N} = 2$ supergravity the global part of the near horizon supergroup of the large black ring is $OSp(4^*|2) \times \mathrm{U}(1)$. At the leading order, the entropy function, c-extremization and Brown-Henneaux approaches are in agreement with each other and with the microscopic results obtained in [11, 12, 29, 30].

The paper is organized as follows. In section 2 we review black ring solution of five dimensional $\mathcal{N} = 2$ supergravity and its near horizon geometry. In section 3 we show the supersymmetry enhancement near the horizon of black ring and determine the global part of the superalgebra. In section 4 we apply the entropy function, c-extremization and Brown-Henneaux formalisms for large black rings where we show that the macroscopic and microscopic entropies of black rings are equal to each other. In appendix A Killing vectors of $AdS_2 \times S^1$ component of the black ring near horizon geometry, used in section 3 are given.

2 $\mathcal{N} = 2$ 5D black rings

In this section we briefly review the $\mathcal{N} = 2$ 5D black ring solution in superconformal formalism. In this approach the symmetry group of supergravity is enlarged to superconformal group which can be reduced to the initial model by imposing a suitable gauge fixing con-

the entropy function formalism is found in [26], which can not be applied in the black ring problem.

[4] One encounters a Similar problem in the case of four dimensional spinning black holes. Applying entropy function formalism for spinning black holes is studied in [27].

[5] The Chern-Simons action is gauge invariant up to a boundary term.

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dition. The supersymmetry variations of field content are independent of the Lagrangian and one can consequently apply these variations at any level of corrections.

2.1 Basic setup

The field content of superconformal gravity are arranged in Weyl, vector and hypermultiplets. The bosonic fields of Weyl multiplet are the vielbein e^a_μ, an auxiliary 2-form field v_{ab} and an auxiliary scalar field D. The bosonic part of each vector multiplet contains a 1-form gauge field A^I and a scalar field X^I, where $I = 1, \ldots, n_v$ labels the gauge group. The hypermultiplet contains scalar fields $\mathcal{A}^i_\alpha$, where $i = 1, 2$ is the SU(2) doublet index and $\alpha = 1, \ldots, 2n$ refers to *USp(2n)* group.

In the off-shell formalism the bosonic part of the action of $\mathcal{N} = 2$ supergravity in five dimensions at the leading order is [36]

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{|g|} \mathcal{L}_0, \tag{2.1}$$

in which

$$\begin{aligned}
\mathcal{L}_0 &= \partial_a \mathcal{A}^i_\alpha \partial^a \mathcal{A}^\alpha_i + (2\nu + \mathcal{A}^2)\frac{D}{4} + (2\nu - 3\mathcal{A}^2)\frac{R}{8} + (6\nu - \mathcal{A}^2)\frac{v^2}{2} + 2\nu_I F^I_{ab} v^{ab} \\
&\quad + \frac{1}{4}\nu_{IJ}(F^I_{ab} F^{J\,ab} + 2\partial_a X^I \partial^a X^J) + \frac{e^{-1}}{24} C_{IJK} \epsilon^{abcde} A^I_a F^J_{bc} F^K_{de}. \qquad (2.2)\\
\mathcal{A}^2 &= \mathcal{A}^i_\alpha \mathcal{A}^\alpha_i, \qquad v^2 = v_{ab} v^{ab} \text{ and} \\
\nu &= \frac{1}{6} C_{IJK} X^I X^J X^K, \qquad \nu_I = \frac{1}{2} C_{IJK} X^J X^K, \qquad \nu_{IJ} = C_{IJK} X^K, \qquad (2.3)
\end{aligned}$$

where C_{IJK} are the intersection numbers of the internal space. The fermion fields are the gravitino ψ^i_μ and the auxiliary Majorana spinor χ^i which are in the Weyl multiplet, the gaugino Ω^{Ii} in the vector multiplet and hyperino ζ^α in the hypermultiplet.

As we are interested in supersymmetric bosonic solutions, in which fermion fields are set to zero and the solution is invariant under supersymmetry variations, we concentrate on the study the bosonic terms of the supersymmetry variations of fermions which are given as follows[6]

$$\begin{aligned}
\delta\psi^i_\mu &= \mathcal{D}_\mu \varepsilon^i + \frac{1}{2} v^{ab} \gamma_{\mu ab} \varepsilon^i - \gamma_\mu \eta^i, \\
\delta\chi^i &= D\varepsilon^i - 2\gamma^c \gamma^{ab} \hat{\mathcal{D}}_a v_{bc} \varepsilon^i + \gamma^{ab} \hat{R}_{ab}(V)^i_j \varepsilon^i - 2\gamma^a \varepsilon^i \epsilon_{abcde} v^{bc} v^{de} + 4\gamma^{ab} v_{ab} \eta^i, \\
\delta\Omega^{Ii} &= -\frac{1}{4}\gamma^{ab} F^I_{ab} \varepsilon^i - \frac{1}{2}\gamma^a \partial_a X^I \varepsilon^i - X^I \eta^i, \\
\delta\zeta^\alpha &= \gamma^a \partial_a \mathcal{A}^\alpha_i - \gamma^{ab} v_{ab} \varepsilon^i \mathcal{A}^\alpha_i + 3\mathcal{A}^\alpha_i \eta^i,
\end{aligned} \tag{2.4}$$

[6] Here $\gamma_{a_1 a_2 \cdots a_m} = \frac{1}{m!}\gamma_{[a_1}\gamma_{a_2} \cdots \gamma_{a_m]}$ which is antisymmetric in all indices. Also the covariant curvature $\hat{R}^{ij}_{\mu\nu}$ is defined by $\hat{R}^{ij}_{\mu\nu} = 2\partial_{[\mu} V^{ij}_{\nu]} - 2V^i_{[\mu\ k} V^{kj}_{\nu]} +$ *fermionic terms*, where V^{ij}_μ is a boson in the Weyl multiplet which is in **3** of the SU(2). For the solution we are going to consider, this term vanishes.

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where $\delta \equiv \bar{\epsilon}^i \mathbf{Q}_i + \bar{\eta}^i \mathbf{S}_i + \xi^a_K \mathbf{K}_a$,[7] and the covariant derivatives are defined by

$$\mathcal{D}_\mu \varepsilon^i = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab} + \frac{1}{2}b_\mu\right) - V_\mu{}^i{}_j \varepsilon^j, \tag{2.5}$$

$$\hat{\mathcal{D}}_\mu v_{ab} = (\mathcal{D}_\mu - b_\mu)\, v_{ab} = \partial_\mu v_{ab} + 2\omega_{[a}{}^c v_{b]c} - b_\mu v_{ab}, \tag{2.6}$$

in which b_μ is a real boson in the Weyl multiplet and is SU(2) singlet [36].

There is a well-known gauge to fix the conformal invariance of the off-shell formalism and reduce the superconformal symmetry to the standard symmetries of five dimensional $\mathcal{N}=2$ supergravity,

$$\mathcal{A}^2 = -2, \qquad b_\mu = 0, \qquad V_\mu^{ij} = 0. \tag{2.7}$$

In this gauge the last equation of (2.4) gives η^i in terms of ε^i as,

$$\eta^i = \frac{1}{3}\gamma^{ab} v_{ab} \varepsilon^i. \tag{2.8}$$

In the gauge (2.7) and also after solving the equation of motion of the auxiliary fields D and v_{ab}, the Lagrangian density (2.2) reduces to the standard form of the bosonic part of $\mathcal{N}=2$ supergravity in five dimensions,

$$\mathcal{L}_0 = R - \frac{1}{2}G_{IJ}F^I_{ab}F^{Jab} - G_{IJ}\partial_a X^I \partial^a X^J + \frac{e^{-1}}{24}C_{IJK}A^I_a F^J_{bc}F^K_{de}\epsilon^{abcde}, \tag{2.9}$$

where

$$G_{IJ} = -\frac{1}{2}\partial_I\partial_J(\ln\nu) = \frac{1}{2}(\nu_I\nu_J - \nu_{IJ}), \tag{2.10}$$

and the supersymmetry variations (2.4) simplify as

$$\begin{aligned}
\delta\psi^i_\mu &= \left(\mathcal{D}_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}\gamma_\mu\gamma^{ab}v_{ab}\right)\varepsilon^i, \\
\delta\chi^i &= \left(D - 2\gamma^c\gamma^{ab}\mathcal{D}_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(\gamma^{ab}v_{ab})^2\right)\varepsilon^i, \\
\delta\Omega^{Ii} &= \left(-\frac{1}{4}\gamma^{ab}F^I_{ab} - \frac{1}{2}\gamma^a\partial_a X^I - \frac{1}{3}X^I\gamma^{ab}v_{ab}\right)\varepsilon^i,
\end{aligned} \tag{2.11}$$

where we have used (2.8). In section 3 we use these results for investigating supersymmetry enhancement near the horizon of black ring solution.

2.2 Black ring solutions

The five dimensional $\mathcal{N}=2$ supergravity have several half-BPS black hole, black string and black ring solutions. Here we review the large black ring solutions following [7]. These solutions have both electric Q^I and magnetic p^I charges. From eleven dimensional supergravity point of view, these charges correspond to the $M2$ and $M5$-branes respectively wrapping nontrivial cycles of the internal space. For simplicity we study the $\mathrm{U}(1)^3$ solution

[7] $\mathbf{Q}_i$ is the generator of $\mathcal{N}=2$ supersymmetry, $\mathbf{S}_i$ is the generator of conformal supersymmetry and $\mathbf{K}_a$ are special conformal boost generators of superconformal algebra [36].

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which is the most symmetric solution. The M-theory configuration corresponding to this solution consists of three M2-branes and three M5-branes oriented as [7]

$$\begin{array}{lllllllll} Q_1 & M2: & 1 & 2 & - & - & - & - & - \\ Q_2 & M2: & - & - & 3 & 4 & - & - & - \\ Q_3 & M2: & - & - & - & - & 5 & 6 & - \\ p_1 & M5: & - & - & 3 & 4 & 5 & 6 & \psi \\ p_2 & M5: & 1 & 2 & - & - & 5 & 6 & \psi \\ p_3 & M5: & 1 & 2 & 3 & 4 & - & - & \psi \end{array} \tag{2.12}$$

where directions z_i, $i = 1 \cdots 6$, span the internal 6-torus and ψ is the ring direction of black ring.

The $11D$ supergravity solution takes the form

$$\begin{aligned} ds_{11}^2 &= ds_5^2 + X^1(dz_1^2 + dz_2^2) + X^2(dz_3^2 + dz_4^2) + X^3(dz_5^2 + dz_6^2), \\ \mathcal{A} &= A^1 \wedge dz_1 \wedge dz_2 + A^2 \wedge dz_3 \wedge dz_4 + A^3 \wedge dz_5 \wedge dz_6, \end{aligned} \tag{2.13}$$

where $\mathcal{A}$ is the three-form potential with four-form field strength $\mathcal{F} = d\mathcal{A}$.

The five dimensional solution is specified by a metric ds_5^2, three scalars X^I, and three one-forms A^I, with field strengths $F^I = dA^I$.[8] In ring coordinates the solution is written as follows[9]

$$\begin{aligned} ds_5^2 &= (H_1H_2H_3)^{-2/3}(dt+\omega)^2 - (H_1H_2H_3)^{1/3}d\mathbf{x}_4^2, \\ d\mathbf{x}_4^2 &= \frac{R^2}{(x-y)^2}\left[(y^2-1)d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right], \\ A^I &= H_I^{-1}(dt+\omega) + \frac{p^I}{2}[(1+y)d\psi + (1+x)d\phi], \\ X^I &= H_I^{-1}(H_1H_2H_3)^{1/3}. \end{aligned} \tag{2.14}$$

In these coordinates, $y = -\infty$ corresponds to the location of the ring, and Q^I and p^I are the electric and magnetic charges respectively. The harmonic functions H_I are defined by[10]

$$H_1 = 1 + \frac{Q_1 - p_2p_3}{2R^2}(x-y) - \frac{p_2p_3}{4R^2}(x^2-y^2), \tag{2.15}$$

and the same for H_2 and H_3 with cyclic permutation. For simplicity we choose

$$Q_1 = Q_2 = Q_3 = Q, \qquad p_1 = p_2 = p_3 = p. \tag{2.16}$$

The one-form ω which is related to the angular momentum of the solution is $\omega = \omega_\psi d\psi + \omega_\varphi d\varphi$ with

$$\begin{aligned} \omega_\psi &= \frac{p}{8R^2}(y^2-1)[3Q - p^2(3+x+y)] - \frac{3p}{2}(1+y), \\ \omega_\varphi &= \frac{p}{8R^2}(1-x^2)[3Q - p^2(3+x+y)]. \end{aligned} \tag{2.17}$$

[8] All the fields are independent from internal space and exterior derivative d on A^I is defined in five dimensional space.

[9] for metrics, we use $(+----)$ signature.

[10] $R = 0$ reduces the black ring solution to the BMPV solution [37] although the limit of $R \to 0$ in (2.15) is singular .

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The ADM charges of this solution are given by

$$\begin{aligned} M &= \frac{3\pi}{4G_5}Q, \\ J_\psi &= \frac{\pi}{8G_5}p[6R^2+3Q-p^2], \qquad J_\varphi = \frac{\pi}{8G_5}p(3Q-p^2). \end{aligned} \tag{2.18}$$

The coordinate ranges are

$$-\infty \le y \le 1, \qquad -1 \le x \le 1, \qquad 0 \le \psi \le 2\pi, \qquad 0 \le \varphi \le 2\pi. \tag{2.19}$$

To make the above solution free of closed causal curves for $y \geq -\infty$, one requires that,

$$2p^2L^2 \equiv 2\sum_{i<j}\mathcal{Q}_i p_i \mathcal{Q}_j p_j - \sum_i \mathcal{Q}_i^2 p_i^2 - 2R^2p^3\sum_i p_i \geq 0, \tag{2.20}$$

where

$$p = (p_1p_2p_3)^{1/3}, \qquad \mathcal{Q}_1 = Q_1 - p_2p_3, \qquad \mathcal{Q}_2 = Q_2 - p_1p_3, \qquad \mathcal{Q}_3 = Q_3 - p_1p_2. \tag{2.21}$$

For (2.16),

$$L = \sqrt{3\left[\frac{(Q-p^2)^2}{4p^2} - R^2\right]}, \tag{2.22}$$

There is a nice review on this solution by Emparan and Reall [12].

2.3 Near horizon geometry

In [6, 21] it is shown that in a comfortable coordinate system the near horizon limit of the black ring solution (2.14) becomes,

$$ds^2 = -p^2\left(\frac{dr^2}{4r^2} + \frac{L^2}{p^2}d\psi^2 + \frac{Lr}{p}dtd\psi\right) - \frac{p^2}{4}(d\theta^2 + \sin^2\theta d\phi^2), \tag{2.23}$$

which is the product of a locally AdS_3 with radius p and an S^2 with radius $\frac{p}{2}$. This metric can be written as follows,

$$ds^2 = \frac{p^2}{4}\left(r^2dt^2 - \frac{dr^2}{r^2}\right) - L^2\left(d\psi + \frac{pr}{2L}dt\right)^2 - \frac{p^2}{4}\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2.24}$$

where the range of coordinates are

$$0 \le r \le \infty, \qquad 0 \le \theta \le \pi, \qquad 0 \le \phi \le 2\pi, \qquad 0 \le \psi \le 2\pi. \tag{2.25}$$

The near horizon geometry (2.24) is $AdS_2 \times S^1 \times S^2$ in which the $S^1 \times AdS_2$ component, locally looks like the BTZ black hole with radius $r_+ = L$. Furthermore, in these coordinates the near horizon limit of the field strengths of the gauge fields A^I (2.14) are

$$F^I_{\theta\phi} = -\frac{p^I}{2}\sin\theta, \tag{2.26}$$

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and the attractor values of scalars X^I are

$$X^I = \frac{p^I}{(\frac{1}{6}C_{IJK}p^Ip^Jp^K)^{1/3}}. \tag{2.27}$$

As is well-known, one can in principle obtain considerable information about a black object by studying the corresponding near horizon geometry. In the next two sections we study the enhancement of supersymmetry near the horizon and apply the entropy function, the c-extremization and Brown-Henneaux formalisms to obtain the global supergroup of the near horizon geometry and the entropy of black ring.

3 Enhancement of supersymmetry

The bosonic supersymmetric solution of any supersymmetric theory should be invariant under supersymmetry variations of fermions. Thus both fermions and their supersymmetry variations should be equal to zero.

For studying enhancement of supersymmetry near the horizon of any half-BPS black object one should check if it is possible to make supersymmetry variations of fermions of the theory equal to zero without imposing any additional condition on the Killing spinor ε^i. In our case the supersymmetry variations of auxiliary Majorana spinor χ^i and gaugino Ω^{Ii} in (2.11) give no constraint on spinor ε^i, but imposes the following constraints on the bosonic auxiliary fields,

$$F^I_{\mu\nu} = -\frac{4}{3}X^I v_{\mu\nu}, \qquad D = \frac{8}{3}v^2 \qquad \epsilon^{abcd}\mathcal{D}_a v_{bc} = 0, \qquad \mathcal{D}^b v_{ab} - \frac{1}{3}\epsilon_{abcd}v^{bc}v^{de} = 0. \tag{3.1}$$

Thus we only investigate the gravitino variation in ψ^i (2.11) for supersymmetry enhancement.

3.1 Killing spinor

The calculations will be easier in non-coordinate basis. The components of vielbein for the near horizon geometry of the black ring solution (2.24) are

$$e^{\hat{t}} = \frac{pr}{2}dt, \quad e^{\hat{r}} = \frac{p}{2r}dr, \quad e^{\hat{\theta}} = \frac{p}{2}d\theta, \quad e^{\hat{\phi}} = \frac{p\sin\theta}{2}d\phi, \quad e^{\hat{\psi}} = L\left(d\psi + \frac{pr}{2L}dt\right), \tag{3.2}$$

and the inverse components are given by

$$e^{\hat{t}t} = \frac{2}{pr}, \quad e^{\hat{r}r} = -\frac{2r}{p}, \quad e^{\hat{\theta}\theta} = -\frac{2}{p}, \quad e^{\hat{\phi}\phi} = -\frac{2}{p\sin\theta}, \quad e^{\hat{\psi}\psi} = -\frac{1}{L}, \quad e^{\hat{t}\psi} = -\frac{1}{L}. \tag{3.3}$$

By using the explicit relation for spin connection,

$$(\omega_\mu)^{ab} = \frac{1}{2}e^{\nu a}(\partial_\mu e^b_\nu - \partial_\nu e^b_\mu) - \frac{1}{2}e^{\nu b}(\partial_\mu e^a_\nu - \partial_\nu e^a_\mu) - \frac{1}{2}e^{\rho a}e^{\sigma b}(\partial_\rho e_{\sigma c} - \partial_\sigma e_{\rho c})e^c_\mu, \tag{3.4}$$

one can show that the non zero components of spin connections are

$$(\omega_t)^{\hat{t}\hat{r}} = -\frac{r}{2}, \quad (\omega_t)^{\hat{r}\hat{\psi}} = \frac{r}{2}, \quad (\omega_r)^{\hat{t}\hat{\psi}} = \frac{1}{2r}, \quad (\omega_\phi)^{\hat{\theta}\hat{\phi}} = \cos\theta, \quad (\omega_\psi)^{\hat{t}\hat{r}} = \frac{L}{p}. \tag{3.5}$$

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Supersymmetry variation of gravitino in (2.11) for our background (2.24) and (2.26) simplifies to

$$\delta\psi^i_\mu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + v_{\theta\phi}\gamma_\mu{}^{\theta\phi} - \frac{2}{3}\gamma_\mu\gamma^{\theta\phi}v_{\theta\phi}\right)\varepsilon^i. \tag{3.6}$$

By using the attractor value of the scalars (2.27) together with the value of field strengths (2.26) and the first equality in (3.1) one obtains,

$$v_{\hat\theta\hat\phi} = \frac{3}{2p}. \tag{3.7}$$

Now, setting all the components of gravitino variation (3.6) equal to zero gives the following equations,

$$\begin{aligned}
\nabla_t\varepsilon^i &= \left(\partial_t + \frac{r}{4}\gamma^{\hat t\hat r}(1-\gamma^{\hat r\hat\theta\hat\phi}) - \frac{r}{4}\gamma^{\hat\psi\hat r}(1-\gamma^{\hat r\hat\theta\hat\phi})\right)\varepsilon^i = 0,\\
\nabla_r\varepsilon^i &= \left(\partial_r - \frac{1}{4r}(\gamma^{\hat r\hat\theta\hat\phi} + \gamma^{\hat t\hat\psi})\right)\varepsilon^i = 0,\\
\nabla_\theta\varepsilon^i &= \left(\partial_\theta - \frac{1}{2}\gamma^{\hat\phi}\right)\varepsilon^i = 0,\\
\nabla_\phi\varepsilon^i &= \left(\partial_\phi + \frac{1}{2}\cos\theta\gamma^{\hat\theta\hat\phi} + \frac{1}{2}\sin\theta\gamma^{\hat\theta}\right)\varepsilon^i = 0,\\
\nabla_\psi\varepsilon^i &= \left(\partial_\psi - \frac{L}{2p}(\gamma^{\hat t\hat r} + \gamma^{\hat\theta\hat\phi\hat\psi})\right)\varepsilon^i = 0.
\end{aligned} \tag{3.8}$$

One can easily show that integrability condition,

$$[\nabla_\mu, \nabla_\nu]\varepsilon^i = 0, \tag{3.9}$$

is automatically satisfied without imposing any projection on the Killing spinor ε^i. Thus all the supersymmetry is restored near the horizon.

Assuming $\gamma^{\hat t\hat\psi\hat r\hat\theta\hat\phi} = 1$ equations (3.8) simplify as

$$\begin{aligned}
\nabla_t\varepsilon^i &= \left(\partial_t + \frac{r}{2}(\gamma^{\hat t\hat r} - \gamma^{\hat\psi\hat r})\right)\varepsilon^i = 0,\\
\nabla_r\varepsilon^i &= \left(\partial_r - \frac{1}{2r}\gamma^{\hat t\hat\psi}\right)\varepsilon^i = 0,\\
\nabla_\theta\varepsilon^i &= \left(\partial_\theta - \frac{1}{2}\gamma^{\hat\phi}\right)\varepsilon^i = 0,\\
\nabla_\phi\varepsilon^i &= \left(\partial_\phi + \frac{1}{2}\cos\theta\gamma^{\hat\theta\hat\phi} + \frac{1}{2}\sin\theta\gamma^{\hat\theta}\right)\varepsilon^i = 0,\\
\nabla_\psi\varepsilon^i &= \partial_\psi\varepsilon^i = 0.
\end{aligned} \tag{3.10}$$

There are two solutions corresponding to the projections $\gamma^{\hat r\hat\theta\hat\phi}\varepsilon^i_{(\pm)} = \pm\varepsilon^i_{(\pm)}$. For $\gamma^{\hat r\hat\theta\hat\phi}\varepsilon^i = \varepsilon^i$

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the above equations simplify to,

$$
\begin{aligned}
\nabla_t \varepsilon^i &= \partial_t \varepsilon^i = 0, \\
\nabla_r \varepsilon^i &= (\partial_r - \frac{1}{2r})\varepsilon^i = 0, \\
\nabla_\theta \varepsilon^i &= \left(\partial_\theta - \frac{1}{2}\gamma^{\hat{\phi}}\right)\varepsilon^i = 0, \\
\nabla_\phi \varepsilon^i &= \left(\partial_\phi + \frac{1}{2}\cos\theta\gamma^{\hat{\theta}\hat{\phi}} + \frac{1}{2}\sin\theta\gamma^{\hat{\theta}}\right)\varepsilon^i = 0, \\
\nabla_\psi \varepsilon^i &= \partial_\psi \varepsilon^i = 0.
\end{aligned} \tag{3.11}
$$

It is easy to show that there are two solutions for these equations,

$$
\varepsilon^i = \sqrt{\frac{r}{l}} e^{\frac{1}{2}\gamma^{\hat{\phi}}\theta} e^{-\frac{1}{2}\gamma^{\hat{\theta}\hat{\phi}}\phi}\varepsilon_0^i, \qquad \lambda^i = l\left(-t + \frac{1}{r}\gamma^{\hat{t}\hat{r}}\right)\varepsilon^i, \tag{3.12}
$$

where $l = p/2$ is the radius of both AdS_2 and S^2 part of the near horizon geometry (2.24) and ε_0^i is a constant spinor satisfying $\gamma^{\hat{r}\hat{\theta}\hat{\phi}}\varepsilon_0^i = \varepsilon_0^i$. The relation between two different chiralities is $\varepsilon_{0(-)}^i = \gamma^{\hat{t}\hat{r}}\varepsilon_{0(+)}^i$.

It is enlightening to note that the Killing spinors solution (3.12) depends only on the radius of AdS_2 and S^2 which is proportional to the cube root of the central charge, $c = 6p^3$ (4.9).

3.2 Near horizon superalgebra

As it is shown in appendix A the isometries of $AdS_2 \times S^1$ are generated by,

$$
\begin{aligned}
K_1 &= \frac{1}{2}(t^2 + r^{-2})\partial_t - rt\partial_r - \frac{p}{2Lr}\partial_\psi, & K_2 &= t\partial_t - r\partial_r, \\
K_3 &= \partial_\psi, & K_4 &= \partial_t, \\
K_5 &= e^{\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t + r\partial_r - \frac{p}{2L}\partial_\psi\right), & K_6 &= e^{-\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t - r\partial_r - \frac{p}{2L}\partial_\psi\right).
\end{aligned} \tag{3.13}
$$

The algebra associated to these isometries is as follows

$$
\begin{aligned}
[K_1, K_2] &= -K_1, & [K_1, K_4] &= -K_2, & [K_2, K_4] &= -K_4, \\
[K_3, K_5] &= \frac{2L}{p}K_5, & [K_3, K_6] &= -\frac{2L}{p}K_6, & [K_5, K_6] &= -\frac{p}{L}K_3.
\end{aligned} \tag{3.14}
$$

Using the following redefinitions,

$$
\begin{aligned}
pK_1 &\to L_1^+, & \frac{2}{p}K_4 &\to L_{-1}^+, & -K_2 &\to L_0^+, \\
K_5 &\to L_1^-, & K_6 &\to L_{-1}^-, & -\frac{p}{2L}K_3 &\to L_0^-,
\end{aligned} \tag{3.15}
$$

the algebra (3.14) simplifies to,

$$
[L_m^\pm, L_n^\pm] = (m-n)L_{m+n}^\pm, \qquad m, n = \pm 1, 0. \tag{3.16}
$$

This may be identified to the left and right moving global parts of the CFT_2 which is dual to this locally AdS_3 geometry. Since the generators K_5 and K_6 defined in eq. (3.13) are not single valued functions of $\psi \sim \psi + 2\pi$ the $\mathrm{SL}(2)_L$ is broken to U(1) generated by K_3. Consequently the global symmetry of the near horizon geometry is $\mathrm{U}(1)_L \times \mathrm{SL}(2)_R$.

The action of these generators on the spinors ε^i and λ^i (3.12) can be defined by the Lie derivative $\mathcal{L}_K \varepsilon^i = (K^\mu \mathcal{D}_\mu + \frac{1}{4}\partial_\mu K_\nu \gamma^{\mu\nu})\varepsilon^i$, which gives,

$$\begin{aligned} &L^+_{-1}\lambda^i = -\,\varepsilon^i, && L^+_0\lambda^i = -\frac{1}{2}\lambda^i, && L^+_0\varepsilon^i = \frac{1}{2}\varepsilon^i, && L^+_1\varepsilon^i = \lambda^i,\\ &L^+_{-1}\varepsilon^i = 0, && L^+_1\varepsilon^i = 0, && L^-_m\varepsilon^i = 0, && L^-_m\lambda^i = 0, \end{aligned} \tag{3.17}$$

Considering a correspondence between ε^i and λ^i and the $G_{-\frac{1}{2}}$ and $G_{\frac{1}{2}}$ modes of supercurrent G then (3.17) simplify to

$$[L^+_m, G_r] = \left(\frac{m}{2} - r\right) G_{m+r}, \qquad [L^-_m, G_r] = 0. \tag{3.18}$$

To complete the algebra, we should also study the behavior of the Killing spinors (3.12) under the generators of S^2. These generators are

$$J^3 = -i\partial_\phi, \qquad J^\pm = e^{\pm i\phi}(-i\partial_\theta \pm \cot\theta\,\partial_\phi). \tag{3.19}$$

Since $\gamma^{\hat{r}\hat{\theta}\hat{\phi}}$ and $\gamma^{\hat{\theta}\hat{\phi}}$ commute with each other one can choose

$$\gamma^{\hat{\theta}\hat{\phi}}\varepsilon^i_0 = \pm i\varepsilon^i_0, \tag{3.20}$$

which gives

$$J^3\varepsilon^i = \pm\frac{1}{2}\varepsilon^i, \qquad J^3\lambda^i = \pm\frac{1}{2}\lambda^i. \tag{3.21}$$

Thus both ε^i and λ^i are in the **2** representation of the SU(2) group which is generated by J^is. If one starts with a constant spinor which satisfies $\gamma^{\hat{\theta}\hat{\phi}}\varepsilon^i_0 = -i\varepsilon_0$,[11] and define

$$\xi_+ = \sqrt{\frac{r}{l}}\, e^{\frac{1}{2}\gamma^{\hat{\phi}}\theta} e^{\frac{i}{2}\phi}\varepsilon_0, \qquad \xi_- = \sqrt{\frac{r}{l}}\, e^{\frac{1}{2}\gamma^{\hat{\phi}}\theta} e^{-\frac{i}{2}\phi}\gamma^{\hat{\theta}}\varepsilon_0, \tag{3.22}$$

one verifies that ξ^a are in the **2** representation of the SU(2) group, $J^\pm_0\xi^a_\pm = 0$ and $J^\pm_0\xi^a_\mp = \xi^a_\pm$.

Our results so far can be organized into symplectic-Majorana killing spinors[12]

$$\varepsilon^1 = \begin{pmatrix}\xi_+\\ i\xi_-\end{pmatrix}, \quad \varepsilon^2 = \begin{pmatrix}-i\xi_+\\ -\xi_-\end{pmatrix}, \quad \varepsilon^3 = \begin{pmatrix}\xi_-\\ -i\xi_+\end{pmatrix}, \quad \varepsilon^4 = \begin{pmatrix}i\xi_-\\ -\xi_+\end{pmatrix}, \tag{3.23}$$

where each ε^I transforms as **2** of Sp(2) and corresponds to $G^I_{-\frac{1}{2}}$. In the same manner one can define

$$\lambda^I = l\left(-t + \frac{1}{r}\gamma^{\hat{t}\hat{r}}\right)\varepsilon^I \tag{3.24}$$

[11] Normalization is $\varepsilon_0^\dagger\varepsilon_0 = 1/2$.

[12] symplectic-Majorana condition is $\overline{\zeta}^i = \zeta_i^\dagger\gamma^{\hat{t}} = \zeta^{iT}C$.

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which corresponds to $G^I_{\frac{1}{2}}$ and also transforms as **2** of Sp(2).

To complete the near horizon superalgebra we need to compute the anticommutators of supercharges. To do this we use the supersymmetry transformations of the five dimensional supergravity given by [36, 38, 39],

$$\begin{aligned}\{G^I_r, G^J_s\} &= l\Omega_{ij}\left[(\bar{\varepsilon}^I_r)^i\gamma^\mu(\varepsilon^J_s)^j + (\bar{\varepsilon}^J_s)^i\gamma^\mu(\varepsilon^I_r)^j\right]\partial_\mu \\ &\quad + \left[(\bar{\varepsilon}^I_r)_i\gamma^{\hat{\theta}\hat{\phi}}(\varepsilon^J_s)^j + (\bar{\varepsilon}^J_s)_i\gamma^{\hat{\theta}\hat{\phi}}(\varepsilon^I_r)^j\right],\end{aligned} \tag{3.25}$$

where Ω_{ij} is a symplectic matrix which can be used for raising and lowering the indices as follows,

$$\chi^i = \Omega^{ij}\chi_j, \qquad \chi_i = \chi^j\Omega_{ji}, \tag{3.26}$$

and we have chosen a basis $\Omega_{12} = 1$. By plugging the supercharges (3.23) and (3.24) into (3.25) we derive the anticommutators of the supercharges,

$$\left\{G^I_{\pm\frac{1}{2}}, G^J_{\pm\frac{1}{2}}\right\} = -2\delta^{IJ}L^+_{\pm}, \tag{3.27}$$

and

$$\left\{G^I_{-\frac{1}{2}}, G^J_{\frac{1}{2}}\right\} = \begin{pmatrix} -2L_0^+ & 2iJ^3 + i\sigma_3 & 2iJ^2 + i\sigma_1 & -2iJ^1 + i\sigma_2 \\ -2iJ^3 - i\sigma_3 & -2L_0^+ & -2iJ^1 - i\sigma_2 & -2iJ^2 + i\sigma_3 \\ -2iJ^2 - i\sigma_1 & 2iJ^1 + i\sigma_2 & -2L_0^+ & 2iJ^3 - i\sigma_3 \\ 2iJ^1 - i\sigma_2 & 2iJ^2 - i\sigma_1 & -2iJ^3 + i\sigma_3 & -2L_0^+ \end{pmatrix}, \tag{3.28}$$

where $m, n = 0, \pm1$, $r, s = \pm\frac{1}{2}$, $I, J = 1, 2, 3, 4$ and σ_a are the Pauli matrices. We can summarize all the results to the following superalgebra,

$$\begin{aligned}\{G^I_r, G^J_s\} &= -2\delta^{IJ}L^+_{r+s} + (r-s)(M_a)^{IJ}J^a + (r-s)(N_A)^{IJ}T^A, \\ \left[L^+_m, L^+_n\right] &= (m-n)L^+_{m+n}, \qquad [L^+_m, G^I_r] = \left(\frac{m}{2} - r\right)G^I_{m+r}, \\ \left[J^\alpha, G^I_r\right] &= (t^\alpha)^{IJ}G^J_r, \qquad [T^A, G^I_r] = (N^A)^{IJ}G^J_r,\end{aligned} \tag{3.29}$$

where M_a and N_A are the representation matrices for SU(2) and Sp(2), respectively and T^A are generators of Sp(2). Therefore the bosonic part of the global supergroup is

$$\mathrm{U}(1)_L \times \mathrm{SL}(2)_R \times \mathrm{SU}(2) \times \mathrm{Sp}(2), \tag{3.30}$$

while the isometry of the near horizon geometry of the black ring solution is $\mathrm{U}(1)_L \times \mathrm{SL}(2)_R \times \mathrm{SU}(2)$.[13] The generators in (3.29) are null under the $\mathrm{U}(1)_L$ generated by L_0^- given in (3.15). The extra Sp(2) in (3.30) can be identified with R-symmetry of $\mathcal{N} = 2$ supergravity in five dimensions [14, 16].

Searching in the literature (for example see [42]) we found that there is a supergroup with this bosonic part and supporting eight supercharges which is $D(2,1;1) = Osp(4^*|2)$. So we propose that (3.29) correspond to the $Osp(4^*|2) \times \mathrm{U}(1)$.

[13] The symmetries of the near horizon geometry of the extremal black ring and four dimensional spinning black holes are studied for example in [40] and [41] respectively.

It is interesting to note that this supergroup is the same as the small black string near horizon supergroup [14]. Of course, in [14] the superalgebra of small black string in $\mathcal{N}=4$ five dimensional supergravity is calculated by embedding the solution of $\mathcal{N}=2$ supergravity. For this solution, the supergroup of near horizon is $Osp(4^*|4)\times \mathrm{U}(1)$, where the $Osp(4^*|4)$ part of superalgebra is

$$\begin{aligned}
\{G^I_r, G^J_s\} &= -2\delta^{IJ}L_{r+s} + (r-s)(t_\alpha)^{IJ}J^\alpha + (r-s)(\rho_A)^{IJ}R^A,\\
[L_m, L_n] &= (m-n)L_{m+n}, \qquad [L_m, G^I_r] = \left(\frac{m}{2}-r\right)G^I_{m+r},\\
[J^\alpha, G^I_r] &= (t^\alpha)^{IJ}G^J_r, \qquad [R^A, G^I_r] = (\rho^A)^{IJ}G^J_r,
\end{aligned} \tag{3.31}$$

in which t^α and ρ^A are the representation matrices for SU(2) and Sp(4) respectively and R^A are the generators of Sp(4). In [16], it was shown that this global part of superalgebra in $\mathcal{N}=4$ supergravity is reduced to $Osp(4^*|2)$ in $\mathcal{N}=2$.[14] This result shows that in AdS/CFT analysis black ring solution behaves like a small black string.

It is straightforward to repeat the above calculations when higher order corrections are considered, as higher order corrections only modify p and L in the metric (2.24) [43]–[47]. Therefore, after adding e.g. the supersymmetric correction [36], the supersymmetry is still enhanced near the horizon and the superalgebra does not change.

4 Near horizon physics

A special feature of the supersymmetric black ring is the geometry of the near horizon of this solution such that an $AdS_2\times S^1$ is locally AdS_3 (2.23)–(2.24). This special near horizon topology allows one to apply both the entropy function [20] and the c-extremization [28] formalisms on black ring. We also use Brown-Henneaux approach [32] to calculate the CFT entropy of extremal black ring.

4.1 Entropy function

In this section we briefly review the entropy function formalism applied to the black ring solution to calculate the corresponding macroscopic entropy [21–23].

In the entropy function formalism the entropy can be found from the extremum of the entropy function,

$$S = 2\pi(e^I q_I - f), \tag{4.1}$$

in which,

$$f = \int dx_H \sqrt{|g_H|}\mathcal{L}, \tag{4.2}$$

[14] It is interesting to note that $Osp(4^*|2)$ factor is also present in the small black string [14], the small black hole [16] and the large black ring solutions (3.29) of $\mathcal{N}=2$ supergravity in five dimensions.

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and q_I is defined by $q_I = \frac{\partial f}{\partial e^I}$. To apply the entropy function for the near horizon geometry of the black ring one uses the ansatz [21][15]

$$\begin{aligned} ds^2 &= v_1(-r^2dt^2 + \frac{dr^2}{r^2}) + v_2(d\theta^2 + \sin^2\theta d\phi^2) + w(d\psi^2 + e_0 r dt)^2, \\ F^I_{5rt} &= e^I + a^I e_0, \qquad F^I_{5\theta\phi} = \frac{p^I}{2}\sin\theta, \qquad X^I = M^I, \qquad I = 1,2,3, \end{aligned} \tag{4.3}$$

where e_0 is conjugate to the angular momentum of the ring. Extremizing the entropy function (4.1) with respect to the v_1, v_2, w, M^I and N^I gives,

$$v_1 = v_2 = \frac{p^2}{4}, \qquad w = \frac{p}{2e_0}, \qquad e^I + e_0 N^I = 0, \qquad M^I = \frac{p^I}{(\frac{1}{6}C_{IJK}p^Ip^Jp^K)^{1/3}}. \tag{4.4}$$

Using (4.4) and (4.1) one obtains,

$$S_{\rm mac} = 2\pi p^2 L. \tag{4.5}$$

The same result is obtained in the off-shell formalism in [22].

4.2 c-extremization

In [28] Kraus and Larsen showed that for D-dimensional black objects with $AdS_3 \times S^{D-3}$ near horizon geometry one can define the c-function as[16]

$$c(l_A, l_S) = \frac{3\Omega_2\Omega_{D-3}}{32\pi G_D} l_A^3 l_S^{D-3}\mathcal{L}, \tag{4.6}$$

which extremization with respect to the radii of AdS_3 and S^2, gives the average of the left and right central charges of CFT dual of AdS_3. As we have discussed in section 2 and as can also be verified using the Killing vectors derived in the appendix A, the near horizon geometry of the black ring solution, has locally an AdS_3 component (2.23). Thus one can expect that c-extremization formalism [28] can also be applied for black ring solution.

We consider the following ansatz,

$$\begin{aligned} ds^2 &= l_A^2 d\Omega^2_{AdS_3} + l_S^2 d\Omega^2_{S^2}, \qquad X^I = m p^I, \\ F^I_{rt} &= e^I + e_0 a^I, \qquad F^I_{\theta\phi} = \frac{p^I}{2}\sin\theta, \qquad v_{rt} = v_1 v_{\theta\phi} = v_2 \sin\theta. \end{aligned} \tag{4.7}$$

After solving the equations of motion of D, v_{ab}, m and a^I one finds[17]

$$D = \frac{12}{p^2}, \qquad m = p^{-1}, \qquad v_2 = -\frac{3}{8}p, \qquad v_1 = 0, \qquad e^I + e_0 a^I = 0, \tag{4.8}$$

where $p \equiv (\frac{1}{6}C_{IJK}p^Ip^Jp^K)^{1/3}$.[18] By extremizing the c-function one obtains,

$$l_A = 2l_S = p, \qquad c = 6p^3. \tag{4.9}$$

[15] In this section following the usual conventions in both the entropy function and the c-extremization formalisms we use $(-++++)$ signature and choose $G_5 = \pi/4$.

[16] It was discussed in [28] that only the bulk part of the action contributes in this definition.

[17] These results can also be derived from the supersymmetry variations of fermions (2.11) [22].

[18] In $U(1)^3$ supergravity which is the subject of our study in this paper, all p^I are equal to each other and denote them by p.

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In the semiclassical regime $c \gg 1$, $c \sim c_L \sim c_R$, in which $c_{L(R)}$ is the left (right) central charge of the CFT. In this limit, $(c_L - c_R)$ is negligible as it is given by higher order corrections, and $c = \frac{1}{2}(c_L + c_R)$ is given by the c-extremization method (4.9). Since the black ring solution in $\mathcal{N} = 2$ five dimensional supergravity corresponds to the $(0, 4)$-CFT the microscopic entropy is given by the logarithm of the number of left moving excitations [11, 12, 29, 30],[19]

$$S_{\rm mic} = 2\pi\sqrt{\frac{c\hat{q}_0}{6}}, \tag{4.10}$$

where [12]

$$\begin{aligned}\hat{q}_0 &= \frac{p_1 p_2 p_3}{4} + \frac{1}{2}\left(\frac{Q_1 Q_2}{p_3} + \frac{Q_2 Q_3}{p_1} + \frac{Q_3 Q_1}{p_2}\right) \qquad (4.11)\\ &\quad + \frac{1}{4p_1 p_2 p_3}[(p_1 Q_1)^2 + (p_2 Q_2)^2 + (p_3 Q_3)^2] - J_\psi \\ &= pL^2. \qquad (4.12)\end{aligned}$$

Thus, at the leading order, the result of the c-extremization formalism (4.9) and microscopic description of the entropy of the black ring are in agreement with the entropy calculated by the macroscopic entropy function formalism (4.5),

$$S_{\rm mac} = S_{\rm mic} = 2\pi L p^2. \tag{4.13}$$

4.3 Brown-Henneaux approach

In this section we recalculate the microscopic entropy of supersymmetric black rings from another viewpoint by using the Kerr/CFT formalism [31]. In this method, which is intrinsically a generalization of the Brown-Henneaux approach [32], the Virasoro generators of the CFT dual are related to the asymptotic symmetry group (ASG) of the near horizon metric. The asymptotic symmetry group (ASG) of a near horizon metric is the group of allowed symmetries modulo trivial symmetries. By definition, an allowed symmetry transformation obeys the specified boundary conditions [31]. A possible boundary condition for the fluctuations around the geometry (2.24) is,

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} r^2 & 1/r^2 & 1/r & r & 1 \\ & 1/r^3 & 1/r^2 & 1/r^2 & 1/r \\ & & 1/r & 1/r & 1/r \\ & & & 1/r & 1 \\ & & & & 1 \end{pmatrix}, \tag{4.14}$$

in the basis $(t, r, \theta, \phi, \psi)$. It is easy to show that the general diffeomorphism preserving the boundary conditions (4.14) is given by,

$$\begin{aligned}\zeta = &\left[C + \mathcal{O}\left(\frac{1}{r^3}\right)\right]\partial_t + \left[r\epsilon'(\psi) + \mathcal{O}(1)\right]\partial_r + \mathcal{O}\left(\frac{1}{r}\right)\partial_\theta \\ &+\mathcal{O}\left(\frac{1}{r^2}\right)\partial_\phi + \left[\epsilon(\psi) + \mathcal{O}\left(\frac{1}{r^2}\right)\right]\partial_\psi, \qquad (4.15)\end{aligned}$$

[19] We are using the conventions of [29] for left and right moving central charges.

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where C is an arbitrary constant and $\epsilon(\psi)$ is the arbitrary smooth periodic functions of ψ. By using the basis $\epsilon_n(\psi) = -e^{-in\psi}$ for the function $\epsilon(\psi)$, it is easy to show that the ASG generates contains a Virasoro algebra generated by

$$\zeta_n = -e^{-in\psi}\partial_\psi - in\ r\ e^{-in\psi}\partial_r, \tag{4.16}$$

which satisfy $[\zeta_m, \zeta_n] = -i(m-n)\zeta_{n+n}$.

The generator of a diffeomorphism has a conserved charge. The charges associated to the diffeomorphisms (4.16) are defined by [34],

$$Q_\zeta = \frac{1}{8\pi}\int_{\partial\Sigma} k_\zeta[h, g], \tag{4.17}$$

where $\partial\Sigma$ is spatial surface at infinity and

$$\begin{aligned} k_\zeta[h, g] = \frac{1}{2}\Big[&\zeta_\nu\nabla_\mu h - \zeta_\nu\nabla_\sigma h_\mu{}^\sigma + \zeta_\sigma\nabla_\nu h_\mu{}^\sigma + \frac{h}{2}\nabla_\nu\zeta_\mu \\ &- h_\nu{}^\sigma\nabla_\sigma\zeta_\mu + \frac{1}{2}h_{\nu\sigma}(\nabla_\mu\zeta^\sigma + \nabla^\sigma\zeta_\mu)\Big] * (dx^\mu \wedge dx^\nu), \end{aligned} \tag{4.18}$$

in which $*$ denotes the Hodge dual in 5D. In the Brown-Henneaux approach [32] the central charge is given by

$$\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n}, g] = -\frac{i}{12}c(m^3 - m)\delta_{m+n,0}. \tag{4.19}$$

Plugging the metric (2.24) and diffeomorphisms (4.16) in (4.19) one obtains,

$$c = 6p^3, \tag{4.20}$$

which is in agreement with the c-extremization result (4.9).

The Frolov-Thorne temperature can be determined by identifying quantum numbers of a matter field in the near horizon geometry with those in original geometry. For the chiral CFT given by (4.16) a matter field can be expanded in eigen modes of the asymptotic energy ω and angular momentum m as

$$\Phi = \sum_{\omega,m,l}\varphi_{\omega m l}e^{-i(\omega t - m\psi)}f_l(r, \theta, \phi), \tag{4.21}$$

Similar to [31] one can show that, here, the Frolov-Thorne temperature is

$$T_{\rm FT} = \frac{1}{2\pi e_0}. \tag{4.22}$$

The Cardy formula gives the microscopic entropy of chiral CFT (4.16) as follows,

$$S_{\rm CFT} = \frac{\pi^2}{3}c\ T_{\rm FT} = 2\pi L p^2, \tag{4.23}$$

which is in precise agreement with the result obtained by utilizing the entropy function and the c-extremization methods (4.13).

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A Killing vectors of $AdS_2 \times S^1$ geometry

In this appendix we derive the Killing vectors of $AdS_2 \times S^1$ geometry which is appeared in the near horizon of black ring solution of $\mathcal{N} = 2$ five dimensional supergravity. The metric of this part is (2.23),

$$ds^2 = -p^2 \left(\frac{dr^2}{4r^2} + \frac{L^2}{p^2} d\psi^2 + \frac{Lr}{p} d\psi dt \right), \tag{A.1}$$

and Killing equation is

$$X^\rho \partial_\rho g_{\mu\nu} + \partial_\mu X^\rho g_{\rho\nu} + \partial_\nu X^\rho g_{\mu\rho} = 0. \tag{A.2}$$

The components of Killing equation are

$$\partial_t X^\psi = 0, \tag{A.3}$$
$$\partial_t X^r + \frac{2Lr^3}{p} \partial_r X^\psi = 0, \tag{A.4}$$
$$X^r + r\partial_t X^t + r\partial_\psi X^\psi = 0, \tag{A.5}$$
$$X^r - r\partial_r X^r = 0, \tag{A.6}$$
$$\partial_\psi X^r + \frac{2Lr^3}{p} \partial_r X^t + \frac{4L^2 r^2}{p^2} \partial_r X^\psi = 0, \tag{A.7}$$
$$\partial_\psi X^t + \frac{2L}{pr} \partial_\psi X^\psi = 0. \tag{A.8}$$

Equations (A.3) and (A.6) show that,

$$X^\psi = f(r, \psi), \qquad X^r = rg(t, \psi). \tag{A.9}$$

So we can simplify (A.3)–(A.8) to obtain,

$$\partial_t g(t, \psi) + \frac{2Lr^2}{p} \partial_r f(r, \psi) = 0, \tag{A.10}$$
$$g(t, \psi) + \partial_t X^t + \partial_\psi f(r, \psi) = 0, \tag{A.11}$$
$$\partial_\psi g(t, \psi) + \frac{2Lr^2}{p} \partial_r X^t + \frac{4L^2 r}{p^2} \partial_r f(r, \psi) = 0, \tag{A.12}$$
$$\partial_\psi X^t + \frac{2L}{pr} \partial_\psi f(r, \psi) = 0. \tag{A.13}$$

In (A.10) the first term is a function of t and ψ and the second term is a function of r and ψ. Therefore, each term only is a function of ψ,

$$\partial_t g(t, \psi) = -\frac{2Lr^2}{p} \partial_r f(r, \psi) = h(\psi), \tag{A.14}$$

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and consequently,

$$g_(t,\psi) = h(\psi)t + g_1(\psi), \qquad f(r,\psi) = \frac{p}{2Lr}h(\psi) + f_1(\psi). \tag{A.15}$$

Now we can simplify (A.11)–(A.13) as,

$$h(\psi)t + g_1(\psi) + \partial_t X^t + \frac{p}{2Lr}\partial_\psi h(\psi) + \partial_\psi f_1(\psi) = 0, \tag{A.16}$$

$$\partial_\psi h(\psi)t + \partial_\psi g_1(\psi) + \frac{2Lr^2}{p}\partial_r X^t - \frac{2L}{pr}h(\psi) = 0, \tag{A.17}$$

$$\partial_\psi X^t + \frac{1}{r^2}\partial_\psi h(\psi) + \frac{2L}{pr}\partial_\psi f_1(\psi) = 0. \tag{A.18}$$

From (A.16) one finds

$$X^t = -\left(\frac{1}{2}h(\psi)t^2 + g_1(\psi)t + \frac{p}{2Lr}\partial_\psi h(\psi)t + \partial_\psi f_1(\psi)t\right) + I(r,\psi). \tag{A.19}$$

(A.19) and (A.17) give

$$2\partial_\psi h(\psi)t + \partial_\psi g_1(\psi) + \frac{2Lr^2}{p}\partial_r I(r,\psi) - \frac{2L}{pr}h(\psi) = 0. \tag{A.20}$$

This implies that,

$$\partial_\psi h(\psi) = 0 \qquad \Rightarrow \qquad h(\psi) = c_1. \tag{A.21}$$

Thus (A.15) simplifies as

$$g_(t,\psi) = c_1 t + g_1(\psi), \qquad f(r,\psi) = \frac{p}{2Lr}c_1 + f_1(\psi), \tag{A.22}$$

and (A.16)–(A.19) become

$$c_1 t + g_1(\psi) + \partial_t X^t + \partial_\psi f_1(\psi) = 0, \tag{A.23}$$

$$\partial_\psi g_1(\psi) + \frac{2Lr^2}{p}\partial_r X^t - \frac{2L}{pr}c_1 = 0, \tag{A.24}$$

$$\partial_\psi X^t + \frac{2L}{pr}\partial_\psi f_1(\psi) = 0, \tag{A.25}$$

$$X^t = -\left(\frac{1}{2}c_1 t^2 + g_1(\psi)t + \partial_\psi f_1(\psi)t\right) + I(r,\psi). \tag{A.26}$$

By (A.26), eqs. (A.24) and (A.25) simplify to

$$\partial_\psi g_1(\psi) + \frac{2Lr^2}{p}\partial_r I(r,\psi) - \frac{2L}{pr}c_1 = 0, \tag{A.27}$$

$$-\left(\partial_\psi g_1(\psi) + \partial_\psi^2 f_1(\psi)\right)t + \partial_\psi I(r,\psi) + \frac{2L}{pr}\partial_\psi f_1(\psi) = 0. \tag{A.28}$$

From (A.27) one obtains

$$I(r,\psi) = -\frac{1}{2r^2}c_1 + \frac{p}{2Lr}\partial_\psi g_1(\psi) + I_1(\psi), \tag{A.29}$$

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and so (A.28) becomes

$$-t\left(\partial_\psi g_1(\psi)+\partial_\psi^2 f_1(\psi)\right)+\frac{p}{2Lr}\partial_\psi^2 g_1(\psi)+\partial_\psi I_1(\psi)+\frac{2L}{pr}\partial_\psi f_1(\psi)=0, \tag{A.30}$$

which implies that,

$$\begin{aligned}\partial_\psi g_1(\psi)+\partial_\psi^2 f_1(\psi) &= 0, && \text{(A.31)}\\ \frac{p}{2L}\partial_\psi^2 g_1(\psi)+\frac{2L}{p}\partial_\psi f_1(\psi) &= 0, && \text{(A.32)}\\ \partial_\psi I_1(\psi) &= 0. && \text{(A.33)}\end{aligned}$$

Thus,

$$\begin{aligned}g_1(\psi)+\partial_\psi f_1(\psi) &= c', && \text{(A.34)}\\ \partial_\psi g_1(\psi)+\frac{4L^2}{p^2}f_1(\psi) &= c_3, && \text{(A.35)}\\ I_1(\psi) &= c_4. && \text{(A.36)}\end{aligned}$$

Now we can simplify our results. From (A.34) and (A.35) one obtains,

$$-\partial_\psi^2 f_1(\psi)+\frac{4L^2}{p^2}f_1(\psi)=c_3 \Rightarrow f_1(\psi)=c_5 e^{\frac{2L}{p}\psi}+c_6 e^{-\frac{2L}{p}\psi}+\frac{p^2}{4L^2}c_3, \tag{A.37}$$

Therefore,

$$g_1(\psi)=c_2+\frac{2L}{p}(c_6 e^{-\frac{2L}{p}\psi}-c_5 e^{\frac{2L}{p}\psi}). \tag{A.38}$$

Alternatively we can solve $g_1(\psi)$ as,

$$-\partial_\psi^2 g_1(\psi)+\frac{4L^2}{p^2}g_1(\psi)=\frac{4L^2}{p^2}c_2 \Rightarrow g_1(\psi)=c_7 e^{\frac{2L}{p}\psi}+c_8 e^{-\frac{2L}{p}\psi}+c_2, \tag{A.39}$$

which is consistent with the first solution. c_8.

In summary,

$$\begin{aligned}I(r,\psi) &= -\frac{1}{2r^2}c_1-\frac{2L}{pr}\left(c_5 e^{\frac{2L}{p}\psi}+c_6 e^{-\frac{2L}{p}\psi}\right)+c_4, && \text{(A.40)}\\ g(t,\psi) &= c_1 t+c_2+\frac{2L}{p}(c_6 e^{-\frac{2L}{p}\psi}-c_5 e^{\frac{2L}{p}\psi}), && \text{(A.41)}\\ f(r,\psi) &= \frac{p}{2Lr}c_1+c_5 e^{\frac{2L}{p}\psi}+c_6 e^{-\frac{2L}{p}\psi}+\frac{p^2}{4L^2}c_3. && \text{(A.42)}\end{aligned}$$

So,

$$\begin{aligned}X^t &= -\left(\frac{1}{2}c_1 t^2+c_2 t\right)-\frac{1}{2r^2}c_1-\frac{2L}{pr}\left(c_5 e^{\frac{2L}{p}\psi}+c_6 e^{-\frac{2L}{p}\psi}\right)+c_4, && \text{(A.43)}\\ X^r &= r\left(c_1 t+c_2+\frac{2L}{p}(c_6 e^{-\frac{2L}{p}\psi}-c_5 e^{\frac{2L}{p}\psi})\right), && \text{(A.44)}\\ X^\psi &= \frac{p}{2Lr}c_1+c_5 e^{\frac{2L}{p}\psi}+c_6 e^{-\frac{2L}{p}\psi}+\frac{p^2}{4L^2}c_3. && \text{(A.45)}\end{aligned}$$

Thus the killing vector expand as follows,

$$\begin{aligned}
X &= X^t\partial_t + X^r\partial_r + X^\psi\partial_\psi \\
&= -c_1\left(\frac{1}{2}(t^2+r^{-2})\partial_t - rt\partial_r - \frac{p}{2Lr}\partial_\psi\right) \\
&\quad -c_2\left(t\partial_t - r\partial_r\right) + c_3\frac{p^2}{4L^2}\partial_\psi + c_4\partial_t \\
&\quad -c_5\frac{2L}{p}e^{\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t + r\partial_r - \frac{p}{2L}\partial_\psi\right) \\
&\quad -c_6\frac{2L}{p}e^{-\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t - r\partial_r - \frac{p}{2L}\partial_\psi\right),
\end{aligned} \tag{A.46}$$

and consequently, there are six isometries generated by,

$$\begin{aligned}
K_1 &= \frac{1}{2}(t^2+r^{-2})\partial_t - rt\partial_r - \frac{p}{2Lr}\partial_\psi, & K_2 &= t\partial_t - r\partial_r, \\
K_3 &= \partial_\psi, & K_4 &= \partial_t \\
K_5 &= e^{\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t + r\partial_r - \frac{p}{2L}\partial_\psi\right), & K_6 &= e^{-\frac{2L}{p}\psi}\left(\frac{1}{r}\partial_t - r\partial_r - \frac{p}{2L}\partial_\psi\right).
\end{aligned} \tag{A.47}$$

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